MODELING OF CABLE-STAYED BRIDGES FOR ANALYSIS OF TRAFFIC INDUCED VIBRATIONS

Raid Karoumi

Department of Structural Engineering
Royal Institute of Technology
SE-100 44 Stockholm, Sweden
www.struct.kth.se

ABSTRACT

This paper presents a method for modeling and analysis of cable-stayed bridges under the action of moving vehicles. Accurate and efficient finite elements are used for modeling the bridge structure. A beam element is adopted for modeling the girder and the pylons. Whereas, a two-node catenary cable element derived using exact analytical expressions for the elastic catenary, is adopted for modeling the cables. The vehicle model used in this study is a so-called suspension model that includes both primary and secondary vehicle suspension systems. Bridge damping, bridge-vehicle interaction and all sources of geometric nonlinearity are considered. An iterative scheme is utilized to include the dynamic interaction between the bridge and the moving vehicles. The dynamic response is evaluated using the mode superposition technique and utilizing the deformed dead load tangent stiffness matrix. To illustrate the efficiency of the solution methodology and to highlight the dynamic effects, a numerical example of a simple cable-stayed bridge model is presented.

NOMENCLATURE

L_x, L_y, L_u	projected and unstressed lengths of the cable element
$u_1,\cdots,u_4$	horizontal and vertical nodal displacements
$P_1,\cdots,P_4$	horizontal and vertical end forces of the cable element
T_i, T_j	tension forces at the two nodes of the cable element
E	modulus of elasticity
A	cross section area
I	moment of inertia
w	weight per unit length
m	mass
ρ	density
c_p, c_s	damping coefficient of vehicle viscous dampers
k_p, k_s	stiffness of vehicle springs
t	time
v	vehicle speed
ξ	bridge damping ratio
ω	circular frequency
μ	mass ratio
$\mathbf{p}$	internal force vector
$\mathbf{f}$	external force vector
$\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$	nodal displacement, velocity, and acceleration vectors
$\mathbf{M}$	mass matrix
$\mathbf{C}$	damping matrix
$\mathbf{F}$	flexibility matrix
$\mathbf{K}$	stiffness matrix
$\mathbf{K}_t$	tangent stiffness matrix
TMD	tuned mass damper

1. INTRODUCTION

Due to their aesthetic appearance, efficient utilization of structural materials and other notable advantages, cable-stayed bridges have gained much popularity in recent decades. Bridges of this type are now entering a new era with main span lengths reaching 1000 m. This fact is due, on one hand to the relatively small size of the substructures required and on the other hand to the development of efficient construction techniques and to the rapid progress in the analysis and design of this type of bridges.

The recent developments in design technology, material qualities, and efficient construction techniques in bridge engineering enable the construction of not only longer but also lighter and more slender bridges. Thus nowadays, very long span slender cable-stayed bridges are being built, and the ambition is to further increase the span length and use shallower and more slender girders for future bridges. To achieve this, accurate procedures need to be developed that can lead to a thorough understanding and a realistic prediction of the structural response due to not only wind and earthquake loading but also traffic loading. It is well known that large deflections and vibrations caused by dynamic tire forces of heavy vehicles can lead to bridge deterioration and eventually increasing maintenance costs and decreasing service life of the bridge structure.

Although several long span cable-stayed bridges are being build or proposed for future bridges, little is known about their dynamic behavior under the action of moving vehicles. The dynamic response of bridges subjected to moving vehicles is complicated. This is because the dynamic effects induced by moving vehicles on the bridge are greatly influenced by the interaction between the vehicles and the bridge structure. To consider dynamic effects due to moving vehicles on bridges, structural engineers worldwide rely on dynamic amplification factors specified in bridge design codes. These factors are usually a function of the bridge fundamental natural frequency or span length and states how many times the static effects must be magnified in order to cover the additional dynamic loads. This is the traditional method used today for design purpose and can yield a conservative and expensive design for some bridges but might underestimate the dynamic effects for others. In addition, design codes disagree on how this factor should be evaluated and today, when comparing different national codes, a wide range of variation is found for the dynamic amplification factor. Thus, improved analytical techniques that consider all the important parameters that influence the dynamic response are required in order to check the true capacity of existing bridges to heavier traffic and for proper design of new bridges.

The recent developments in bridge engineering have also affected damping capacity of bridge structures. Major sources of damping in conventional bridgework have been largely eliminated in modern bridge designs reducing the damping to undesirably low levels. As an example, welded joints are extensively used nowadays in modern bridge designs. This has greatly reduced the hysteresis that was provided in riveted or bolted joints in earlier bridges. For cable supported bridges and in particular long span cable-stayed bridges, energy dissipation is very low and is often not enough on its own to suppress vibrations. To increase the overall damping capacity of the bridge structure, one possible option is to incorporate external dampers (i.e. discrete damping devices such as viscous dampers and tuned mass dampers) into the system. Such devices are frequently used today for cable supported bridges. However, it is not believed that this is always the most effective and the most economic solution. Therefore, a great deal of research is needed to investigate the damping capacity of modern cable-stayed bridges and to find new alternatives to increase the overall damping of the bridge structure.

In this paper, the linear dynamic response of a simple two-dimensional cable-stayed bridge model, subjected to a moving vehicle, is studied. Bridge damping, exact cable behavior, and nonlinear geometric effects are considered. This study focuses on investigating the influence of vehicle speed, bridge damping, bridge-vehicle interaction, and a tuned mass damper on the bridge dynamic response.

2. BRIDGE AND VEHICLE MODELING

2.1 Bridge Structure

Modern cable-stayed bridges exhibit geometrically nonlinear behavior, they are very flexible and undergo large displacements before attaining their equilibrium configuration. As an example, due to this inherently nonlinear behavior, conventional linear dead load analysis, which assumes small displacements, is often not applicable [1].

Cable-stayed bridges consist of cables, pylons and girders (bridge decks) and are usually modeled using beam and bar elements for the analysis of the global structural response. To consider the nonlinear behavior of the cables, each cable is usually replaced by one bar element with equivalent cable stiffness. This approach is referred to as the equivalent modulus approach and has been used by several investigators, see e.g. [1, 2, 3]. It has been shown in [4] that the equivalent modulus approach results in softer cable response as it accounts for the sag effect but does not account for the stiffening effect due to large displacements. Still, for some cases, e.g. for short span cable-stayed bridges, analysis utilizing the equivalent modulus approach is often sufficient [3], especially in the feasibility design stage. Whereas, long span cable-stayed bridges built today or proposed for future bridges are very flexible, they undergo large displacements, and should therefore be analyzed taking into account all sources of geometric nonlinearity. Although several investigators studied the behavior of cable-stayed bridges, very few tackled the problem of using cable elements for modeling the cables. See ref. [5, 6] where different cable modeling techniques are discussed and references to literature dealing with the analysis and the behavior of cable structures are given.

In this paper, an alternative approach is presented where accurate and efficient elements are adopted for the modeling. A beam element, which includes geometrically nonlinear effects and is derived using a consistent mass formulation, is adopted for modeling the girder and the pylons. Whereas, a two-node cable element derived using exact analytical expressions for the elastic catenary, is adopted for modeling the cables. The nonlinear finite element method is utilized considering all sources of geometric nonlinearity, i.e. change of cable geometry under different tension load levels (cable sag effect), change of the bridge geometry due to large displacements, and axial force-bending moment interaction in the bridge deck and pylons (P- δ effect).

The adopted beam element, able to resist bending, shear, and axial forces, is developed following the total Lagrangian approach and using a linear interpolation scheme for the displacement components. This element is chosen because it can handle large displacements and shear deformations and because it is simple to formulate the element matrices. This beam element is of minor interest and, due to space limitation, not discussed here in more detail. The interested reader is referred to the author's doctoral thesis, reference [5], where formulation of this beam element is presented in detail.

In the following subsection, the cable element matrices will be given in the element local coordinate system. Using this approach, each cable may be represented by a single 2-node finite element, which accurately consider the curved geometry of the cable. Despite the fact that this cable modeling technique has been available for many years it has, at least to the author's knowledge, very seldom been used for analysis of cables in cable-stayed bridges.

2.1.1 Cable Element

Consider an elastic cable element, stretched in the vertical plane as shown in Figure 1, with an unstressed length L_u, modulus of elasticity E, cross section area A, and weight per unit length w

(uniformly distributed along the unstressed length). For the elastic catenary, the exact relations between the element projections and cable force components at the ends of the element are:

$$L_x = -P_1\left(\frac{L_u}{EA} + \frac{1}{w}\ln\frac{P_4 + T_j}{T_i - P_2}\right) \tag{1a}$$

$$L_y = \frac{1}{2EAw}\left(T_j^2 - T_i^2\right) + \frac{T_j - T_i}{w} \tag{1b}$$

where T_i and T_j are the cable tension forces at the two nodes of the element. For the above expressions it is assumed that the cable is perfectly flexible and Hooke's law is applicable to the cable material.

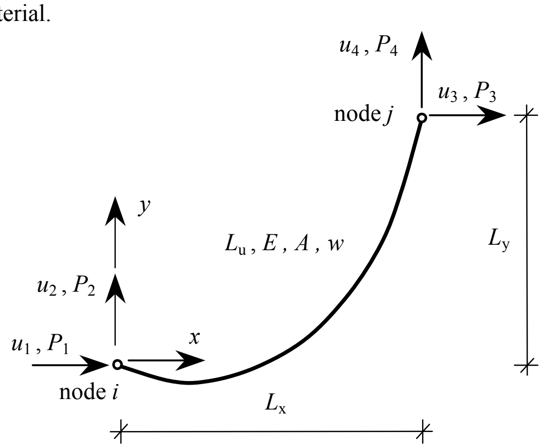

FIGURE 1: Catenary cable element

By rewriting the above expressions for L_x and L_y in terms of the end forces P_1 and P_2 only using the relationships:

$$P_4 = w\,L_u - P_2\ ;\ P_3 = -P_1\ ;\ T_i = \sqrt{P_1^2 + P_2^2}\ ;\ T_j = \sqrt{P_3^2 + P_4^2} \tag{2a-d}$$

differentiating the new expressions for L_x and L_y and rewriting the results using matrix notation gives:

$$\begin{Bmatrix} \mathrm{d}L_x \\ \mathrm{d}L_y \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial L_x}{\partial P_1} & \dfrac{\partial L_x}{\partial P_2} \\ \dfrac{\partial L_y}{\partial P_1} & \dfrac{\partial L_y}{\partial P_2} \end{bmatrix} \begin{Bmatrix} \mathrm{d}P_1 \\ \mathrm{d}P_2 \end{Bmatrix} = \mathbf{F}\begin{Bmatrix} \mathrm{d}P_1 \\ \mathrm{d}P_2 \end{Bmatrix} \tag{3}$$

where $\mathbf{F}$ is the flexibility matrix. The stiffness matrix is given by the inverse of $\mathbf{F}$, i.e. $\mathbf{K} = \mathbf{F}^{-1}$. The tangent stiffness matrix $\mathbf{K_t}$ and the corresponding internal force vector $\mathbf{p}$ for the element can now be obtained in terms of the four nodal degrees of freedom as:

$$\mathbf{K_t} = \begin{bmatrix} -k_1 & -k_2 & k_1 & k_2 \\ & -k_4 & k_2 & k_4 \\ & & -k_1 & -k_2 \\ \text{sym.} & & & -k_4 \end{bmatrix}; \qquad \mathbf{p} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix} \tag{4a,b}$$

where

$$k_1 = -\frac{1}{\det \mathbf{F}}\left(\frac{L_u}{EA} + \frac{1}{w}\left(\frac{P_4}{T_j} + \frac{P_2}{T_i}\right)\right) \tag{5a}$$

$$k_2 = k_3 = -\frac{1}{\det \mathbf{F}}\left(\frac{P_1}{w}\left(\frac{1}{T_j} - \frac{1}{T_i}\right)\right) \tag{5b}$$

$$k_4 = \frac{1}{\det \mathbf{F}}\left(\frac{L_x}{P_1} + \frac{1}{w}\left(\frac{P_4}{T_j} + \frac{P_2}{T_i}\right)\right) \tag{5c}$$

$$\det \mathbf{F} = \left(-\frac{L_u}{EA} - \frac{1}{w}\left(\frac{P_4}{T_j} + \frac{P_2}{T_i}\right)\right)\left(\frac{L_x}{P_1} + \frac{1}{w}\left(\frac{P_4}{T_j} + \frac{P_2}{T_i}\right)\right) - \left(\frac{P_1}{w}\left(\frac{1}{T_j} - \frac{1}{T_i}\right)\right)^2 \tag{5d}$$

The element tangent stiffness matrix $\mathbf{K_t}$ relates the incremental element nodal force vector $\{\Delta P_1, \Delta P_2, \Delta P_3, \Delta P_4\}^T$ to the incremental nodal displacement vector $\{\Delta u_1, \Delta u_2, \Delta u_3, \Delta u_4\}^T$. To evaluate the tangent stiffness matrix $\mathbf{K_t}$, the end forces P_1 and P_2 must be determined first. Those forces are adopted as the redundant forces and are determined, from given positions of cable end nodes, using an iterative stiffness procedure. This procedure requires starting values for the redundant forces. Based on the catenary relationships the following expressions will be used for the starting values:

$$P_1 = -\frac{w\,L_x}{2\lambda} \quad \text{and} \quad P_2 = \frac{w}{2}\left(-L_y\frac{\cosh\lambda}{\sinh\lambda} + L_u\right) \tag{6a,b}$$

where

$$\lambda = \sqrt{3\left(\frac{L_u^2 - L_y^2}{L_x^2} - 1\right)} \tag{6c}$$

In cases where equation (6c) cannot be used because the unstressed cable length is less than the chord length, a conservative value of 0.2 for λ is assumed. Another difficulty arises in equation (6c) for vertical cables. In that case an arbitrary large value of 10^6 for λ is used. Using equations (2a-d), new cable projections corresponding to the assumed end forces P_1 and P_2 are now determined directly from equations (1a,b) and the misclosure vector $\{\Delta L_x\ ,\ \Delta L_y\}^T$ is evaluated as the positions of the end nodes are given. Corrections to the assumed end forces can now be made using the computed misclosure vector as:

$$\begin{Bmatrix} \Delta P_1 \\ \Delta P_2 \end{Bmatrix} = \mathbf{K}\begin{Bmatrix} \Delta L_x \\ \Delta L_y \end{Bmatrix}; \qquad \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}^{i+1} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}^{i} + \begin{Bmatrix} \Delta P_1 \\ \Delta P_2 \end{Bmatrix} \tag{7a,b}$$

where $\mathbf{K}$ is the stiffness matrix (the inverse of $\mathbf{F}$ in equation (3)) and i is the iteration number. For the present study, this iteration process continued until ΔL_X and ΔL_Y are less than 10^{-6}. As will be demonstrated later, this iterative procedure converges very rapidly.

To determine the unstressed cable length, L_u, for cases where the initial cable tension is known instead, a similar iteration procedure can be adopted. A starting value for the unstressed cable length is assumed, e.g. equal to the cable chord length, and cable end forces P_1 and P_2 are computed using the iterative procedure described above. Using equation (2c,d), cable tension can now be computed. This is then compared with the given initial tension to obtain a better approximation for L_u for the next iteration step.

For the dynamic analysis, mass discretization is simply done by static lumping of the element mass at both ends giving the following lumped mass matrix (ρ is the mass density of the cable):

$$\mathbf{M} = \frac{\rho A L_u}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

1.2 Vehicle Model

The vehicle model used in this study is a so-called suspension model that includes both primary and secondary vehicle suspension systems, see Figure 2. This model is sufficient since the main concern is to investigate the dynamic response of bridges and not the dynamics of the vehicle itself and since the spans of cable-stayed bridges are considerably larger than the vehicle axle base. It is assumed that the vehicle never loses contact with the bridge and the contact between the bridge and the moving vehicle is assumed to be a point contact. The equation of motion for the vehicle is coupled to the bridge equation of motion through the interaction force existing at the contact point of the two systems. To solve these two sets of equations, an iterative procedure is adopted, as the interaction force is dependent on the motion of both the bridge structure and the vehicle. Vehicle load modeling and the developed moving load algorithm are described in detail in reference [5]. The implemented codes fully consider the bridge-vehicle dynamic interaction and have been verified in [5].

3. ANALYSIS PROCEDURE

The equation of motion for the entire bridge is obtained as:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K_t}\mathbf{q} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, t) \tag{9}$$

where $\mathbf{q}$, $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$ are the bridge node displacement, velocity, and acceleration vectors, respectively, **M** the bridge mass matrix, **C** the bridge damping matrix, $\mathbf{K_t}$ the tangent stiffness matrix, and $\mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, t)$ the external force vector resulting from the moving vehicles and the tuned mass dampers. As indicated, the external force vector is not only time dependent but is also dependent on the bridge displacements, velocities and accelerations. This vector contains the interaction forces existing at the contact points between the vehicles and the bridge and thereby couples the bridge equation of motion with those of the vehicles.

For this study, the mode superposition technique is adopted utilizing the deformed dead load tangent stiffness matrix ($\mathbf{K_t}$ is obtained from a nonlinear static dead load analysis). It has been shown in [5] that linear dynamic traffic load analysis is adequate for short and medium span cable-stayed bridges as far as traffic load to dead load ratios are small. Moreover, it is well known that the mode superposition technique give sufficiently accurate results with minimum consumption of CPU time, as usually one only need to consider the first dominant modes of vibration. On the other hand, this approach requires frequency analysis and eigenmode extraction to start with, which can be expensive and time consuming for large systems. For the interested reader, details concerning the derivation etc. of this linear dynamic procedure as well as a nonlinear dynamic procedure based on the Newton-Newmark algorithm can be found in [5].

To evaluate the nonlinear static response, an incremental-iterative procedure using full Newton-Raphson iterations is adopted. This procedure is generally expected to give quadratic convergence.

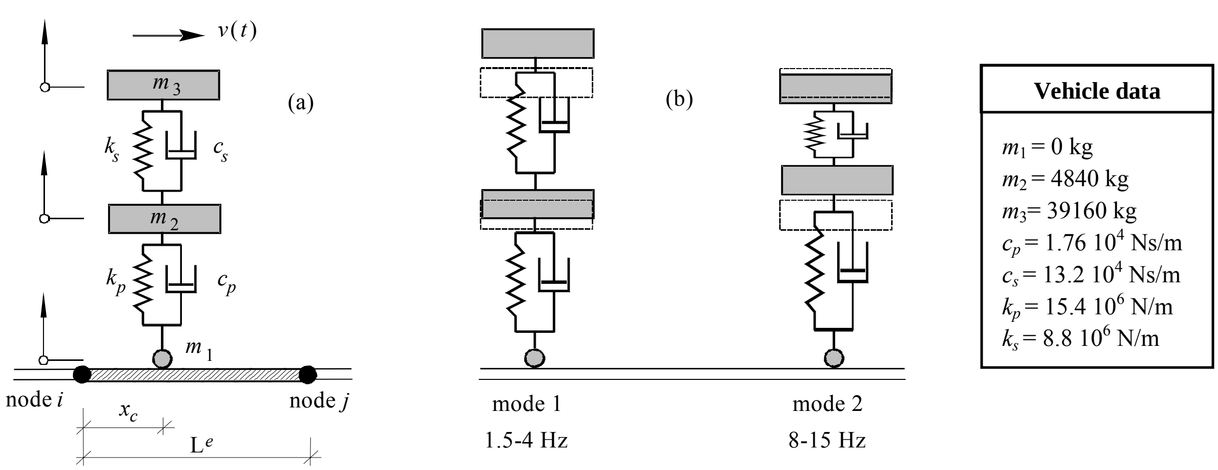

FIGURE 2: (a) Vehicle model on a bridge element; (b) Typical vehicle modes of vibration

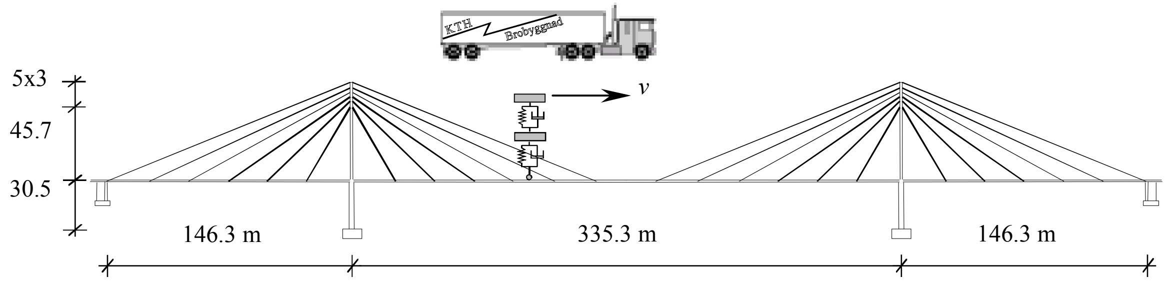

FIGURE 3: Geometry of the cable-stayed bridge

	E (N/m^2)	A (m^2)	I (m^4)	w (t/m)
Girder	$2.0 \cdot 10^{11}$	0.93	0.26	19.64 †
Girder central part	$2.0 \cdot 10^{11}$	1.11	1.29	19.64 †
Pylons above deck level	$2.8 \cdot 10^{10}$	13.01	34.52	30.65
Pylons below deck level	$2.8 \cdot 10^{10}$	18.58	86.31	43.78
Links deck to pylons	$2.0 \cdot 10^{11}$	0.56	0.10	4.38

† Including weight of cross beams.

Cable no.	E (N/m^2)	A (m^2)	L_u (m)	w (t/m)
1, 24	$2.0 \cdot 10^{11}$	0.0362	158.13	0.398
2, 11, 14, 23	$2.0 \cdot 10^{11}$	0.0232	134.66	0.255
3,10, 15, 22	$2.0 \cdot 10^{11}$	0.0204	111.64	0.225
4, 9, 16, 21	$2.0 \cdot 10^{11}$	0.0176	89.43	0.194
5, 8, 17, 20	$2.0 \cdot 10^{11}$	0.0139	68.80	0.153
6, 7, 18, 19	$2.0 \cdot 10^{11}$	0.0113	51.69	0.125
12, 13	$2.0 \cdot 10^{11}$	0.0372	158.12	0.409

TABLE 1: Parameters for the cable-stayed bridge model defined in Figure 3

4. NUMERICAL EXAMPLE

A 2D model of the cable-stayed bridge described in [1] was adopted for this investigation. The bridge geometry is shown in Figure 3 and the properties are given in Table 1.

For the model, it was assumed that the girder was pinned at the ends, i.e. only rotations were allowed, and elastically connected to the pylons by vertical links. The pylons were assumed to be rigidly fixed to the piers, and all cables were assumed fixed to the pylons and to the girder at their joints of attachment. The model had 119 active degrees of freedom and was composed of 66 elements and 43 nodal points. The CPU time used by the computer (200 MHz Pentium Pro) to find the tangent stiffness matrix at the dead load deformed state and solve the system eigenvalue problem determining all 119 modes of vibration, was about 15 seconds. This indicates high efficiency of the presented elements. The first three bending natural frequencies obtained utilizing the dead load tangent stiffness matrix are: 0.332, 0.436, 0.692 Hz. Bridge damping ratios were assumed constant for all modes and equal 0.0056.

This bridge model was then subjected to one and to four 44 ton trucks moving from the left to the right on a smooth road surface at the constant speed v, see Figure 3. The body-bounce and wheel-hop frequencies, for the truck model, were chosen as 1.89 and 11.35 Hz. The corresponding mode shapes and vehicle model properties are shown in Figure 2. To get reasonably converged reliable solutions, the first 30 bridge modes of vibration were considered. In the following subsections, the effect of bridge-vehicle interaction, vehicle speed, bridge damping and a tuned mass damper on the bridge response is presented.

4.1 Effect of Bridge-Vehicle Interaction

A train of four moving 44 ton trucks, 60 m apart, was adopted to investigate the effect of bridge-vehicle interaction. The moving vehicles were modeled either as constant moving forces (i.e. ignoring interaction) or as sprung masses, as in Figure 2, moving on a smooth road surface. 1500 increments corresponding to a time step of 0.021 s were required for this analysis.

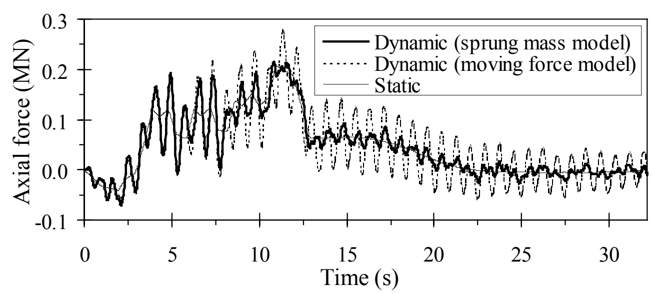

FIGURE 4: Axial force in the shortest cable in side span

Figure 4 above shows the axial force response in the shortest cable in the first side span. Examining this figure it was found that the moving force model, which usually gives negligible differences in results compared to the sprung mass results when the road surface roughness is ignored, gives for this flexible bridge significant differences especially for the tension in this shortest cable. It was also noticed that the dynamic amplification factors (the ratio of the absolute maximum live load dynamic response to the absolute maximum live load static response) decreases for the moving force model except for the tension in this shortest cable where the moving force model increases the dynamic amplification factors tremendously. It is believed that this difference in response occurs as the result of having the vehicles acting as vibration absorbers when modeled as sprung mass systems.

Therefore, as a conclusion, the dynamic interaction between the vehicles and the bridge deck should always be taken into account even if a road surface with no roughness is assumed.

4.2 Vehicle Speed and Bridge Damping Effect

The vertical displacement of the girder at the center of the bridge, due to traffic load only (a single 44 ton moving truck), is shown in Figures 5a and 5b for different speeds and damping ratios. For the curves in Figure 5b, the vehicle speed was $v = 90$ km/h. The static traffic load response is also plotted in this Figure. 1500 increments were required for the solution of the 50 km/h case, and 1000 increments for the rest.

As expected, damping reduces the bridge response. For $\xi = 0.0056$ in Figure 5b, the absolute maximum dynamic displacement is about 20% larger than the static one (dynamic amplification factor of 1.2). It can be concluded from the results in Figure 5 that the response increases with the increase in vehicle speed and that bridge damping has a significant effect upon the response and should therefore always be considered if accurate representation of the true dynamic response is required.

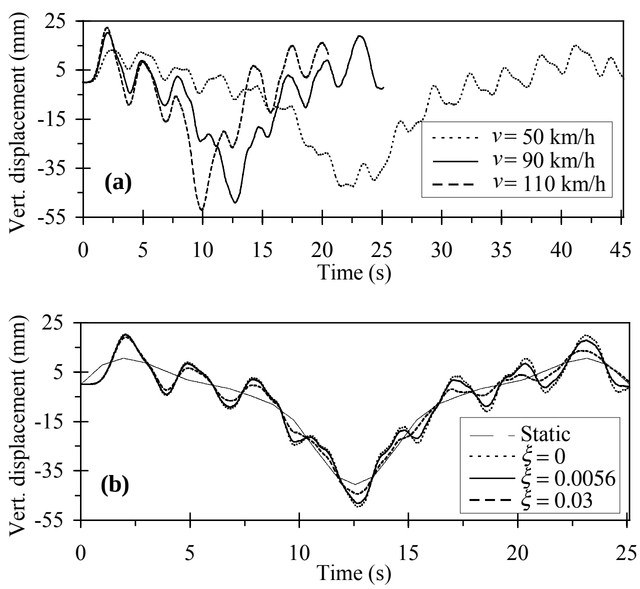

FIGURE 5: Vertical displacement at the center of the bridge calculated for different vehicle speeds (a) and bridge damping ratios (b)

4.3 Effect of a Tuned Mass Damper

The effectiveness of a tuned mass damper (TMD) in suppressing vibrations due to a single 44 ton moving truck is investigated in this study. The truck was assumed to move on a smooth road surface at the constant speed of 110 km/h. Reasonably converged reliable solutions were obtained using 1000 increments corresponding to a time step of 0.025 s. The TMD was positioned at the center of the bridge and tuned to the first bending mode of vibration. The following most often used optimum tuning parameters, derived in [7] for an undamped structure, are adopted:

$$\omega_{tmd} = \frac{\omega_i}{1+\mu} \tag{10a}$$

$$\xi_{tmd} = \sqrt{\frac{3\mu}{8\left(1+\mu\right)^3}} \tag{10b}$$

where ω_{tmd} and ω_i are the circular frequencies of the TMD and the dominant bridge mode to be tuned to, ξ_{tmd} the damping ratio of the TMD, and μ is the mass ratio which relates the TMD mass to the modal mass of the dominant bridge mode to be tuned to, $\mu = m_{tmd} / m_i$. The mass ratio was here set to 0.005 giving a TMD mass of about 15.6 ton. Some results showing the response due to traffic loads only are presented in Figures 7 and 8. Figure 6 shows a cross section of a bridge girder with a tuned mass damper. It was found from the analysis results that the TMD not always is very effective in reducing the maximum dynamic response during the forced vibration period (i.e. when the vehicle is on the bridge). In fact, due to the interaction between the bridge-vehicle-TMD systems, the maximum response for certain elements and nodes can even increase due to the TMD. However, it is evident from Figures 7 and 8 that the TMD is very effective in reducing the vibration level in the free vibration period for all elements and nodes. This is due to the increase of the overall damping of the bridge by the TMD.

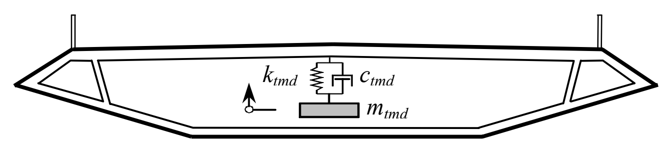

FIGURE 6: Cross section of bridge girder with a TMD

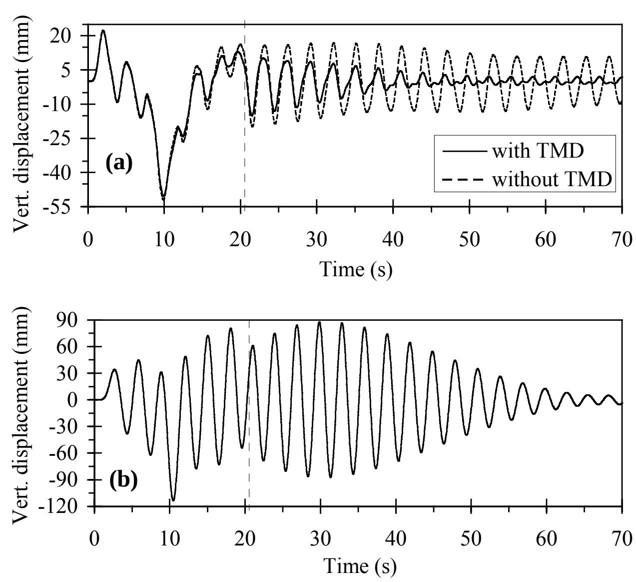

FIGURE 7: (a) Vertical displacement of the girder at the center of the bridge; (b) Vertical displacement of the TMD mass. The dashed vertical line indicate when the truck leaves the bridge

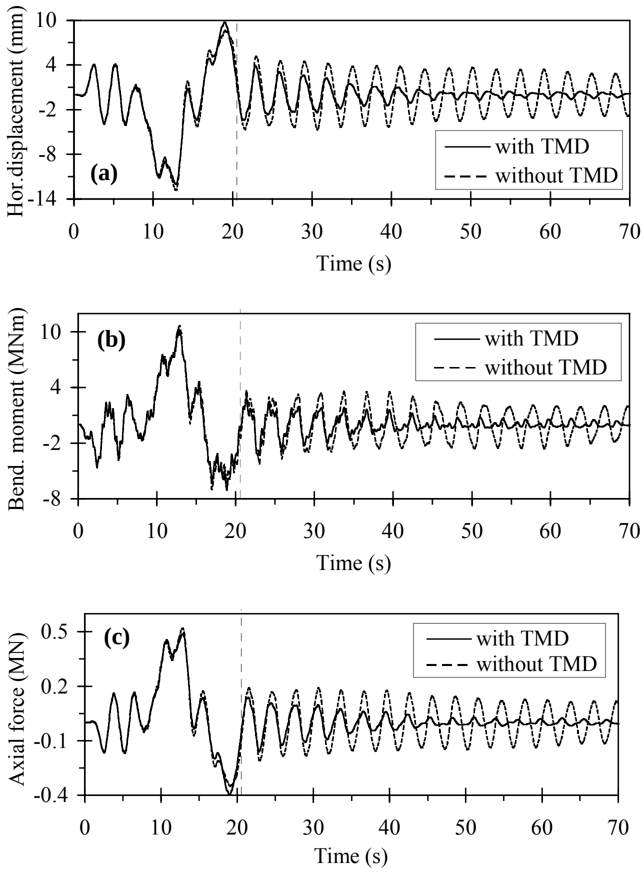

FIGURE 8: (a) Horizontal displacement of the right pylon top; (b) Bending moment at the right pylon fixed end; (c) Axial force in the right anchorage cable. The dashed vertical line indicate when the truck leaves the bridge

5. CONCLUSIONS

This paper has presented a method for modeling and analysis of cable-stayed bridges subjected to moving vehicles. Bridge damping, exact cable behavior, and nonlinear geometric effects have been considered when analyzing the linear dynamic response. The study has only focused on investigating the influence of vehicle speed, bridge damping, bridge-vehicle interaction, and a tuned mass damper on the bridge dynamic response.

A two-node catenary cable element was adopted for modeling the cables and it has been found that the main advantages of this cable element are the simplicity of including the effect of pretension of the cable and the exact treatment of cable sag and cable weight. Moreover, the iterative process adopted to find the internal force vector and tangent stiffness matrix for the cable element is found to converge very rapidly. According to the author's opinion, analysis utilizing the traditional equivalent modulus approach, is not satisfactory for modern cable-stayed bridges. Modern cable-stayed bridges built today or proposed for future bridges are, as they are highly flexible, subjected to large displacements. The equivalent modulus approach however accounts only for the sag effect but not for the stiffening effect due to large displacements.

From the study of the traffic load response of cable-stayed bridges it is concluded that the moving force model (constant force idealization of the vehicle load) can lead to unnecessary overestimation of the dynamic amplification factors compared to the sprung mass model. It has also been shown that the response increases with the increase in vehicle speed and that bridge damping has a significant effect upon the response and should always be considered in such analysis. Bridge damping ratios should be carefully estimated to insure more correct and accurate representation of the true dynamic response. To obtain realistic damping ratios, such estimation should be based on results from tests on similar bridges. Finally, it is concluded that a tuned mass damper is not very effective in reducing the maximum dynamic response during the forced vibration period (i.e. when the vehicle is on the bridge). In fact, such a device can even increase the maximum dynamic response of some nodes and elements. However, the reduction of the vibration level in the free vibration period is significant as the tuned mass damper increases the overall damping of the bridge by working as an additional energy dissipater. The mode superposition technique was found to be very efficient as accurate results could be obtained based on only 30 modes of vibration.

In reference [5], the influence of other important parameters such as road surface roughness, and cables vibration (i.e. multi-element cable discretization) is investigated. In addition, the linear and nonlinear dynamic responses of other bridge models, such as the Great Belt suspension bridge in Denmark, are also studied.

6. REFERENCES

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